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A remark on the glueball-gluon coupling

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Abstract. The QCD trace anomaly motivates the consideration of a low energy glueball-gluon coupling $\phi G_{\mu\nu}{}^a G^{\mu\nu}{}_a$. We point out that this should constitute the leading kinetic term for the gluons at low energies. Anti-screening of the gluons by the glueball then induces a classical Coulomb potential of color charges which increases at large distances $\sim r^{1/3}$ and motivates the inclusion of a corresponding term in the inter-quark potential.

1 Introduction

A peculiar manifestation of the gluon self-interaction in non-abelian gauge theories is the emergence of glueballs in the spectrum [1], and in the present note we would like to reconsider the low energy effective coupling between scalar glueballs and the gluons. If we denote the glueball field by ϕ, a particular coupling term that should arise is $\phi G_{\mu\nu}{}^a G^{\mu\nu}{}_a$, to account for glueball decay. To our knowledge Cornwall and Soni were the first to explicitly propose such a contribution to a low energy effective Lagrangian for QCD [2], and they have also proposed an axion-like coupling of a pseudoscalar glueball. They have derived the coupling terms from sum rules, and in the simplest version one relies on the PCDC type relation for a scalar glueball [3] in the presence of the trace anomaly [4]. The trace anomaly breaks conformal symmetry and gives mass to the glueball, and if the leading contribution of the glueball to the dilatation current in the low energy regime is normalized as $f_\phi \partial_\mu \phi$ the PCDC relation takes the form

$$m^2 f_\phi^2 = -\frac{2\beta(g)}{g} \langle G_{\mu\nu}{}^a G^{\mu\nu}{}_a \rangle. \tag{1}$$

The associated ϕG^2-term entails a mixing of high energy and low energy degrees of freedom similar to the joint appearance of quarks and mesons in chiral quark and quark-meson coupling models [5–15], and the question arises how to avoid overcounting of gluonic degrees of freedom while implementing the glueball-gluon coupling in a low energy effective action for QCD. The solution is simple, yet it bears interesting implications: We propose that the ϕG^2 term should constitute the *leading kinetic term* for the gluons in the low energy regime: Only the kinetic term of the glueballs should survive far away from color sources, while the gluon kinetic term disappears. This in turn implies that the decrease of ϕ with distance increases the chromo-electric potentials proportional to the dynamical permeability $\mu(\phi) = \epsilon(\phi)^{-1} \sim \phi^{-1}$, and this yields an increase of chromo-electric potentials $\sim r^{1/3}$, thus motivating a corresponding contribution to the inter-quark potential. Note that the asymptotic vanishing of the color-singlet dilaton/glueball does not contradict the growth of the chromo-electric potential: We will see below that the chromo-electric field vanishes according to $\mathbf{E}_a \sim \sqrt{\phi} \sim r^{-2/3}$.

The observation of Cornwall and Soni and the argument about the counting of degrees of freedom motivates us to consider the model

$$\mathcal{L} = -\frac{\phi}{4f} G_{\mu\nu}{}^a G^{\mu\nu}{}_a - \frac{1}{2}\partial_\mu\phi \cdot \partial^\mu\phi - \frac{1}{2} m^2 \phi^2 + j_\mu{}^a A^\mu{}_a, \tag{2}$$

where the currents j span an adjoint multiplet of SU(3) and satisfy

$$\partial_\mu j^\mu{}_a + g A_\mu{}^b f_{ab}{}^c j^\mu{}_c = 0. \tag{3}$$

As Cornwall and Soni have pointed out, there also should arise an axion-like coupling of a pseudoscalar glueball, but this plays no role in the examination of the static Coulomb problem below, and therefore we did not include it in (2). We also neglected higher order terms in ϕ and G^2, since these do not affect the asymptotic behavior of the fields at large distances, which is our main concern.

The principles of low energy effective field theories provide a relation between the scales f and f_ϕ in (1) and (2), and the variance of the glueball

$$4\langle \phi^2 \rangle = -\frac{g}{2\beta(g) f} f_\phi^3 - f_\phi^2,$$

but at this point no further determination of f_ϕ is needed.

We have emphasized already that the notion of a glueball-gluon coupling is reminiscent of meson-quark coupling in the chiral quark model, and from the point of view of (1) the glueball in the glueball-gluon coupling model (2) emerges as a QCD dilaton mixing into a gluon bound state.

From the Nambu-Goldstone realization one then might expect a dynamical permittivity $\epsilon(\phi) = \exp(\phi/f)$ like in low energy string or Kaluza-Klein models. The non-abelian Coulomb problem in these models has been solved and yields an ultraviolet regularization $r^{-1} \to (r + r_\phi)^{-1}$ with $r_\phi = g/8\pi\sqrt{3}f$ [16]. However, employing this as a model for a low energy QCD dilaton would imply a distinction between the dilaton and the glueball, because the kinetic terms of the gluons survive in the long wavelength regime: In the model with $\epsilon(\phi) = \exp(\phi/f)$ the $\mathbf{r}$-dependence of ϵ induced by the dilaton in the field of a pointlike color source is

$$\epsilon(\phi(\mathbf{r})) = \exp\Big(\frac{\phi(\mathbf{r})}{f}\Big) = \Big(1 + \frac{r_\phi}{r}\Big)^2.$$

The exponential term also would not properly reproduce the anomaly in the low energy regime, and therefore we focus on the linear coupling (2) in the present paper.

The glueball-gluon coupling thus gives rise to the type of scalar-gluon couplings proposed long ago for classical models of confinement by Kogut and Susskind [17], who considered a $\phi^4 G^2$-term, and by 't Hooft [18]. The present consideration of dynamical permittivites was also motivated by the observation that a $(\phi^2/4f^2)G^2$ term with a scalar of mass m yields a solvable classical Coulomb problem with the chromo-electric potentials of a pointlike charge of color ζ in su(3) given by [19]

$$\Phi = -\frac{\sqrt{3}}{4} f \cdot \Big(\zeta \otimes \zeta^+ - \frac{1}{3}\Big) \cdot \ln\Big(\exp(2mr) - 1\Big).$$

The $\mathbf{r}$-dependence of the dynamical permittivity in this case is

$$\epsilon(\phi(\mathbf{r})) = \frac{\phi^2(\mathbf{r})}{f^2} = \frac{r_\phi}{2m} \frac{1 - \exp(-2mr)}{r^2}.$$

The mechanism of enhancement of the chromo-electric field in these models is similar to confinement from higher order gauge kinetic terms [20–22], where the chromo-electric field itself generates a local permittivity. However, QCD provides a framework for several interesting and promising ideas about confinement: Besides the *leading log* models this includes dual superconductivity from monopoles or vortices, see [23,24] for early references and recent work, and in a different approach Goldhaber and Goldman have pointed out that scalar glueball exchange between quarks yields a confining interaction [25] if the Richardson *ansatz* [26] for the running coupling is employed.

Our concern in the present note is not the identification of the primary confinement mechanism in the strong interactions. Instead, we would like to draw attention to the fact that the glueball-gluon coupling ϕG^2 should be the leading kinetic term for the gluons at low energies, thus motivating the inclusion of an $r^{1/3}$-term in the interquark potential.

2 The classical Coulomb problem in gauge theory with a dynamical permittivity

The glueball-gluon coupling described in the previous section belongs to a class of models which can be parametrized through a field dependent permittivity $\epsilon(\phi)$

$$\mathcal{L} = -\frac{1}{4}\epsilon(\phi) G_{\mu\nu}{}^a G^{\mu\nu}{}_a - \frac{1}{2}\partial_\mu\phi \cdot \partial^\mu\phi - \frac{1}{2}m^2\phi^2 + j_\mu{}^a A^\mu{}_a, \tag{4}$$

and the Coulomb problem for (4) is defined as the problem to determine ϕ and the chromo-electric fields for a static pointlike charge distribution

$$j^\mu{}_a(t, \mathbf{r}) = gC_a\delta(\mathbf{r})\eta^\mu{}_0 \tag{5}$$

from their equations of motion

$$\partial^2\phi = m^2\phi + \frac{1}{4}\epsilon'(\phi) G_{\mu\nu}{}^a G^{\mu\nu}{}_a, \tag{6}$$

$$\partial_\mu\Big(\epsilon(\phi) G^{\mu\nu}{}_a\Big) + g\epsilon(\phi) A_\mu{}^b f_{ab}{}^c G^{\mu\nu}{}_c = -j^\nu{}_a. \tag{7}$$

Here the prime denotes derivation with respect to ϕ. $C_a = \frac{1}{2}\zeta_s^+ \cdot \lambda_a \cdot \zeta_s$ are expectation values of the Gell-Mann matrices $\lambda_a/2$ in color space and satisfy

$$C_a C^a = \frac{1}{3}.$$

The covariant constancy $D_\mu j^\mu = 0$ of the sources follows again from the generalized Yang-Mills equation (7) and $D_\mu D_\nu G^{\mu\nu}{}_a = 0$.

To analyze (6,7) it is convenient to rewrite it in terms of the chromo-electric and magnetic fields $E_i = -G_{0i}{}^a X_a$, $B^i = \frac{1}{2}\epsilon^{ijk} G_{jk}{}^a X_a$:

$$\partial_0^2\phi - \Delta\phi = \frac{1}{2}\epsilon'(\phi)(\mathbf{E}^a \cdot \mathbf{E}_a - \mathbf{B}^a \cdot \mathbf{B}_a) - m^2\phi,$$

$$\nabla \cdot \Big(\epsilon(\phi)\mathbf{E}\Big) - ig\epsilon(\phi)(\mathbf{A} \cdot \mathbf{E} - \mathbf{E} \cdot \mathbf{A}) = \varrho,$$

$$\nabla \cdot \mathbf{B} - ig(\mathbf{A} \cdot \mathbf{B} - \mathbf{B} \cdot \mathbf{A}) = 0,$$

$$\partial_0\Big(\epsilon(\phi)\mathbf{E}\Big) - \nabla \times \Big(\epsilon(\phi)\mathbf{B}\Big)$$

$$+ig\epsilon(\phi)([\Phi, \mathbf{E}] + \mathbf{A} \times \mathbf{B} + \mathbf{B} \times \mathbf{A}) = -\mathbf{j},$$

$$\partial_0\mathbf{B} + ig[\Phi, \mathbf{B}] + \nabla \times \mathbf{E} - ig(\mathbf{A} \times \mathbf{E} + \mathbf{E} \times \mathbf{A}) = 0,$$

where the Bianchi identities were also included and the currents are Lie algebra valued.

For the Coulomb problem this reduces to

$$\nabla \cdot \Big(\epsilon(\phi)\mathbf{E}_a\Big) = gC_a\delta(\mathbf{r}), \tag{8}$$

$$\Delta\phi = m^2\phi - \frac{1}{2}\epsilon'(\phi)\mathbf{E}^a \cdot \mathbf{E}_a, \tag{9}$$

while Faraday's law reduces to $\nabla \times \mathbf{E}_a = 0$ complying with chromo-electric potentials $\mathbf{E}_a = -\nabla\Phi_a$.

Equation (8) yields the chromo-electric potentials

$$\Phi_a(r) = -\frac{gC_a}{4\pi}\int dr\frac{\mu(\phi(r))}{r^2} \qquad (10)$$

where $\mu(\phi) = \epsilon(\phi)^{-1}$ is the dynamical permeability, and this shows that the dynamical permittivity yields confinement if $\lim_{r\to\infty} r\epsilon(\phi(r))$ is bounded.

The scalar field has to be determined from

$$\frac{d^2}{dr^2}r\phi = m^2 r\phi + \frac{\alpha_s\mu'(\phi)}{24\pi r^3}, \qquad (11)$$

and the arguments of Sect. 1 motivate a contribution f/ϕ to μ from the scalar glueball, where f is a mass scale characterizing the strength of the glueball-gluon coupling[1].

The equation for ϕ becomes

$$\frac{d^2}{dr^2}r\phi = m^2 r\phi - \frac{\alpha_s f}{24\pi r^3\phi^2}. \qquad (12)$$

Contrary to the $\phi^2 G^2$ coupling studied in [19], we could find an analytic solution of (12) only for $m = 0$. In this case a first integral to (12) can be found after multiplication by $r^3 d\phi/dr$, and the solution obeying the boundary condition $\lim_{r\to\infty}\phi = 0$ is[2] $\phi^3 = 3\alpha_s f/16\pi r^2$. However, since we are talking about a a low energy effective dynamical permittivity from a scalar dilaton/glueball we are really interested in the behavior for large mr. In that case (12) tells us that

$$\phi \to \Big(\frac{\alpha_s f}{24\pi m^2}\Big)^{1/3} r^{-4/3} \qquad (13)$$

and

$$\Phi_a \to -3C_a\Big(\frac{3g}{2\pi}m^2 f^2 r\Big)^{1/3} \sim -C_a r^{1/3}. \qquad (14)$$

This automatically comes with the right sign for an attractive $q\bar{q}$-interaction in the singlet channel and a repulsive interaction in the octet channel, as well as an attractive qq-interaction for antisymmetric diquark color states and repulsion for symmetric diquark color states: For an (anti-)quark of color ζ_q and a quark of color ζ_s the color factor following from $-C_a$ in (14) in the tensor product basis is

$$C(\zeta_s,\zeta_q) = \pm\frac{1}{2}\Big(|\zeta_s^+\cdot\zeta_q|^2 - \frac{1}{3}\Big) \qquad (15)$$

with the upper sign holding for $q\bar{q}$-interactions and the lower sign applying to quark-quark-interactions. The corresponding color factors in the irreducible representations are then as usual

$$C_1^{q\bar{q}} = \frac{4}{3}$$

for the singlet,

$$C_8^{q\bar{q}} = -\frac{1}{6}$$

for $q\bar{q}$ in the adjoint representation of SU(3),

$$C_S^{qq} = -\frac{1}{3}$$

for the diquarks in symmetric color states, and

$$C_A^{qq} = \frac{2}{3}$$

for the diquarks in anti-symmetric color states.

The static energy density for (2) is

$$\mathcal{H} = \frac{1}{2}\nabla\phi\cdot\nabla\phi + \frac{1}{2}m^2\phi^2 + \frac{\phi}{2f}(\mathbf{E}^a\cdot\mathbf{E}_a + \mathbf{B}^a\cdot\mathbf{B}_a), \qquad (16)$$

and partial integration of the gueball terms yields with (9)

$$H = \frac{3}{4}\int d^3\mathbf{r}\frac{\phi}{f}\mathbf{E}^a\cdot\mathbf{E}_a, \qquad (17)$$

which has the same infrared behavior as

$$-\frac{3}{4}gC^a\int d^3\mathbf{r}'\delta(\mathbf{r}-\mathbf{r}')\Phi_a(\mathbf{r}')$$

and motivates a corresponding power-law term in the inter-quark potential[3].

The glueball-gluon coupling thus motivates a spin-independent contribution to the $q\bar{q}$-potential in the color singlet channel

$$V(r) = \sigma r^{\frac{1}{3}}. \qquad (18)$$

Taking into account that power-law confining potentials with fractional exponents provide good fits to meson spectra and leptonic decay widths [27–32] the emergence of the $r^{1/3}$-term makes it certainly worthwhile to undertake a closer investigation of the corresponding potential model. Dual QCD and the success of linearly increasing

[1] According to (10) we might also incorporate the expected one-gluon exchange term dominating at short distances by adding a constant term:

$$\mu(\phi) = \mu_0 + \frac{f}{\phi}.$$

This would also have the virtue that (4) approaches the underlying QCD Lagrangian in the high energy regime, i.e. close to color sources, while still avoiding overcounting of degrees of freedom in the gluon sector: At high energies the Yang-Mills term approaches the standard kinetic term for the gluons and the scalar decouples, while at low energies no free kinetic term for the gluons exists. We will not include the μ_0 term in the sequel, but we would like to mention that inclusion of such a term would neither affect the behavior of ϕ nor the growth of the chromo-electric potential at large distances.

[2] Superficially the integration of (12) for $m = 0$ yields two integration constants, of which one can be eliminated from absence of a δ-function in the equations for ϕ.

[3] Formally performing a further partial integration in (17) and use of $\nabla\cdot(\epsilon(\phi)\mathbf{E}) = \varrho$ yields again linear coupling between color densities and chromo-electric potentials:

$$H = \frac{3}{4}\int d^3\mathbf{r}\varrho_a\Phi^a.$$

potentials in descriptions of hadron properties [33–36,32] may indicate that the $r^{1/3}$-term provides a subleading contribution both at large and short distances. This might happen, e.g. if monopoles or vortices would dominate at very low energies, but do not fully account for the nonperturbative aspects of the QCD vacuum. Then we might encounter a transition or a crossover from linear confinement to fractional power-law confinement at an intermediate scale, and it would depend on this scale where and when the $r^{1/3}$-term would have an impact on the meson spectrum. Also in this case inclusion of an $r^{1/3}$-term in QCD motivated potentials and comparison with other potential models would provide a test.

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